

ΛΥΣΗ

α) Έχουμε $A = \alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta) = (1 + \sqrt{2} - 1 + \sqrt{2}) \cdot (1 + \sqrt{2} + 1 - \sqrt{2}) = 2\sqrt{2} \cdot 2 = 4\sqrt{2}$.

β) Έχουμε $B = \sqrt{\alpha^2} - \sqrt{\beta^2} = |\alpha| - |\beta| = (1 + \sqrt{2}) - (\sqrt{2} - 1) = 1 + \sqrt{2} - \sqrt{2} + 1 = 2$.

γ) Έχουμε ισοδύναμα:

$$\sqrt{\alpha^2 - \beta^2} > \sqrt{\alpha^2} - \sqrt{\beta^2} \stackrel{\alpha, \beta}{\Leftrightarrow}$$

$$\sqrt{A} > B \Leftrightarrow$$

$$\sqrt{4\sqrt{2}} > 2 \Leftrightarrow$$

$$4\sqrt{2} > 4 \Leftrightarrow$$

$$\sqrt{2} > 1 \Leftrightarrow$$

$2 > 1$, που ισχύει.